NORTH SYDNEY GIRLS HIGH SCHOOL

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

Attempt Questions 1–10
All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top.
Submit one bundle.
The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number: ____________________________ **Teacher:** ___________________________

Student Name: ______________________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
8	/12
9	/12
10	/12
TOTAL	/120

Total Marks – 120
Attempt Questions 1–10
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of $\log_e 7$ correct to 2 decimal places. **1**

(b) Solve $2x+8\le 6$ and graph the solution on a number line. **2**

(c) What is the exact value of $\tan\frac{5\pi}{6}$? **1**

(d) Simplify $\dfrac{x-2}{x+3}\div\dfrac{3x-6}{x^2-x-12}$. **2**

(e) Solve the pair of simultaneous equations: **2**

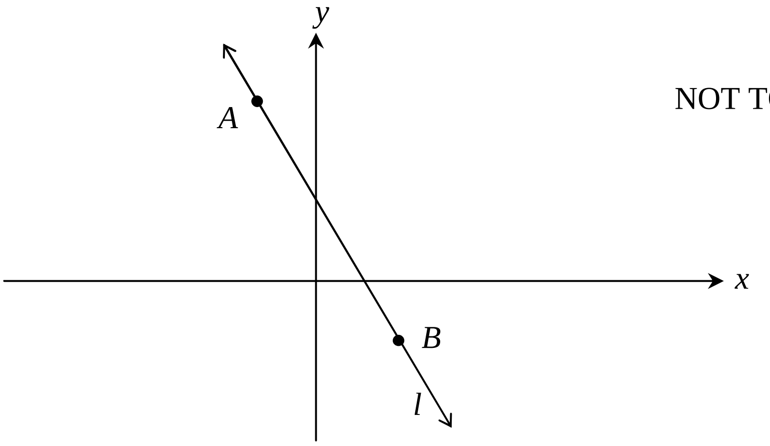
$$x+y=8$$
$$3x-2y=-11$$

(f) Solve $|2x+1|=7$. **2**

(g) Find the values of a and b if $\dfrac{5-\sqrt{3}}{\sqrt{3}-1}=a+b\sqrt{3}$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A is the point $(-1,5)$ and B is the point $(2,-2)$. The line l though A and B has the equation $7x+3y-8=0$.



NOT TO SCALE

(i) State the gradient of the line l. **1**

(ii) Find the angle that the line l makes with the positive x-axis to the nearest degree. **2**

(iii) Find the exact length of the interval AB. **1**

(iv) AC is perpendicular to AB. Find its equation in general form. **2**

(v) A circle with its centre at A is drawn through B. Find the equation of this circle. **1**

(vi) D is the point $(7,-1)$. Find the perpendicular distance from D to the line AB. **2**

(vii) Find the area of the triangle ABD. **1**

(b) Solve $e^{2x}-3e^{x}=4$ giving your answer(s) in exact form. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following with respect to x:

(i) $4x\log_e 2x$ **2**

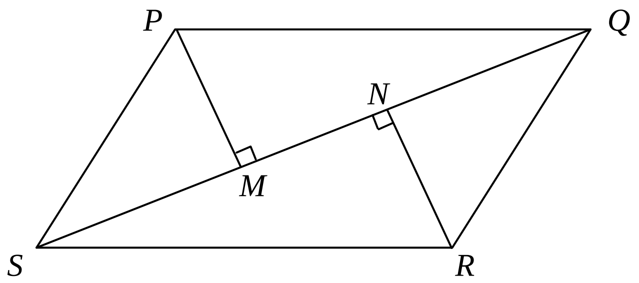
(ii) $\cos(4x-3)$ **1**

(b) The third term of a geometric series is 54 and the sixth term is 2. Find:

(i) the common ratio; **2**

(ii) the sum of the first 6 terms. **2**

(c) *PQRS* is a parallelogram. *PM* and *RN* are perpendicular to *QS*.



NOT TO SCALE

(i) Copy the diagram into your writing booklet.

(ii) Prove that $\Delta MPQ \equiv \Delta NRS$. **3**

(iii) Hence prove that *PNRM* is a parallelogram. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the tangent to the curve $y = 3e^{2x}$ at $(0,3)$. **2**

(b) Find:

(i) $\int \sec^2 4x\, dx$. **1**

(ii) $\int_{-2}^{1} \frac{1}{2-x}\, dx$. **3**

(c) There is an 80% chance that Troy will achieve a Band 6 in Mathematics and a 90% chance that Gabriella will.

(i) Draw a probability tree diagram showing this information. **1**

(ii) What is the chance that only one fails to achieve a Band 6? **1**

(iii) What is the chance that at least one fails to achieve a Band 6? **1**

(d) Determine the value(s) of k for which the expression **3**

$$x^2 + (2-k)x + k(2-k)$$

is positive definite.

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A curve has a gradient function with equation $\frac{dy}{dx} = 6(x-1)(x-2)$.

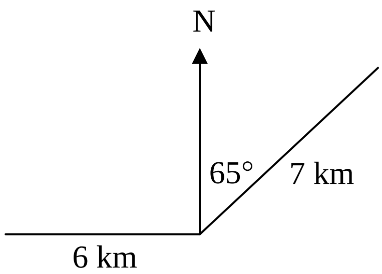
(i) If the curve passes through the point $(1, 2)$, find the equation of the curve. **2**

(ii) Find the coordinates of the stationary points and determine their nature. **3**

(iii) Find any points of inflexion. **2**

(iv) Sketch the graph of the function, showing these key features and the *y* intercept. **2**

(b) An orienteerer hikes 6 km due East. She then turns on a bearing of 065°T and hikes a further 7 km to reach her destination.



NOT TO SCALE

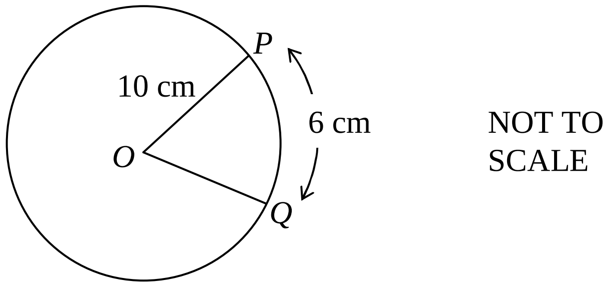
(i) Copy the diagram into your writing booklet.

(ii) Find the length of the shortest possible route back to her starting point, correct to the nearest metre. **2**

(iii) Find the true bearing of her destination from her starting point. **1**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The arc PQ of a circle of radius 10 cm is 6 cm long.



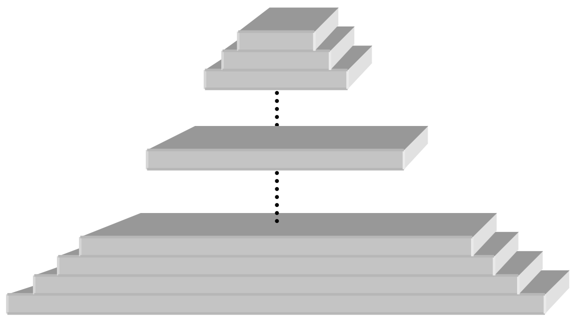
Calculate:

(i) the angle subtended by PQ at the centre O, expressing your answer in degrees correct to the nearest minute; **2**

(ii) the area of the sector POQ; **1**

(iii) the area of the minor segment of the circle cut off by the chord PQ. **1**

(b) A pyramid is built using 1536 blocks on the base level. The next layer contains 1472 blocks and the next 1408, and so on.



(i) How many blocks are used for the ninth layer? **2**

(ii) Before it is capped with a single pyramid block, the top layer has 64 blocks. How many layers are there before the cap is put on? **1**

(iii) How many blocks were used in the construction of the pyramid? **1**

(c) (i) Sketch the curve $y = 3\cos\frac{x}{2}$ for $-\pi \le x \le \pi$. **2**

(ii) Use your graph to determine the number of solutions to the equation $\cos\frac{x}{2} = \dfrac{2x+1}{3}$ that exist in the domain $-\pi \le x \le \pi$. **2**

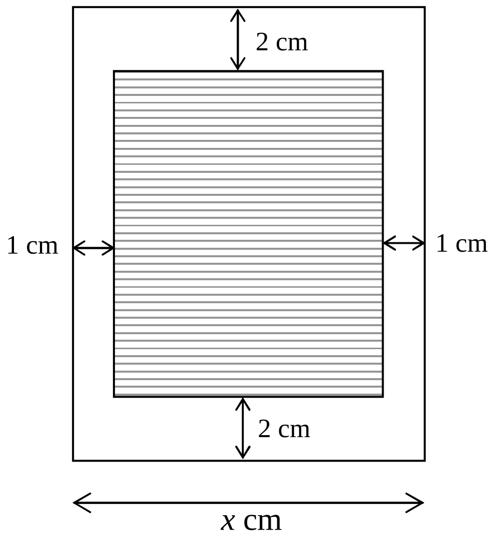
Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Find the points where the line $y = 2x - 3$ intersects the parabola $y = x^2 - 2x - 3$. **2**

(ii) Hence find the exact area enclosed by the line and the parabola. **2**

(b) Solve $2\sec^2 x = 3$ for $0 < x < 2\pi$. **3**
Express your answer in radian measure correct to 2 decimal places.

(c) In a new Mathematics textbook, the pages are to have an area of 338 cm^2. A margin of 1 cm is left at each side and of 2cm at the top and bottom of the page. The width of the page is x cm.



NOT TO SCALE

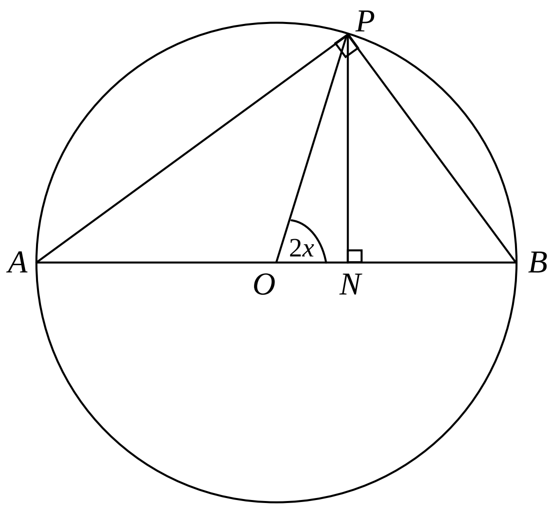
(i) Show that the area A cm^2 of the space available on each for print is given by **2**

$$A = 346 - 4x - \frac{676}{x}.$$

(ii) Hence find the dimensions of the page so that the area of print is maximised. **3**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows a circle with centre O and diameter AB. P is a point on the circumference of the circle. PN is drawn perpendicular to AB and AP is perpendicular to PB. Let $\angle POB = 2x$.



NOT TO SCALE

(i) Explain why $\angle OAP = \angle OPA = x$. **2**

(ii) Show that $\sin 2x = \dfrac{2PN}{AB}$. **2**

(iii) Use ΔAPN and ΔPAB to show that $\sin 2x = 2\sin x \cos x$. **2**

(b) A parabola has the equation $2y = x^2 - 8x + 4$.

(i) Find the coordinates of the vertex. **2**

(ii) State the coordinates of the focus and the equation of the directrix. **2**

(iii) Find the x intercepts of the parabola. **1**

(iv) Hence sketch the parabola. **1**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $\log_b a \times \log_c b \times \log_a c$. **1**

(b) A continuous function is defined by the following features: **3**

$$\frac{d^2y}{dx^2} > 0 \text{ for } x < -1 \text{ and } 1 < x < 3.$$

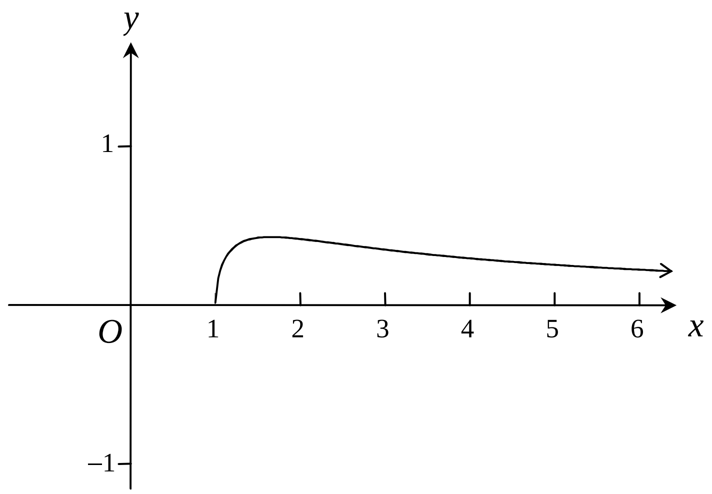
$$\frac{dy}{dx} = 0 \text{ only when } x = -3, 1 \text{ and } 5.$$

$$y = 0 \text{ only when } x = 1.$$

Sketch a possible graph of the function.

(c) Use Simpson's rule with three function values to estimate $\int_1^3 \log_{10} x\,dx$. Give your answer correct to three significant figures. **3**

(d) (i) Differentiate $\dfrac{\log_e x}{x}$. **2**



(ii) The curve $y = \dfrac{\sqrt{\log_e x}}{x}$ in the domain $1 \le x \le e$ is rotated about the x-axis. Using the result in (i), find the volume of the solid formed. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Show that the second derivative of $\log_e(1+\sin x)$ is $-\dfrac{1}{1+\sin x}$. **3**

(b) In January 2000, Judy took a \$300 000 home loan, with interest at 6·0% per annum, compounding monthly.

Judy makes monthly repayments at the end of each month. Let A_n be the amount owing on the loan at the end of each month.

(i) If the monthly repayment is \$2000, show that the amount owing after k months is given by $100000\left[4-(1\cdot005)^k\right]$. **3**

(ii) How much of the loan is still to be repaid after 9 years? **1**

(iii) Find the number of payments Judy will make to pay off the loan. **1**

In January 2009, Judy was unable to make repayments due to the Global Financial Crisis. Her bank offered her a repayment free period of 18 months, during which time interest continued to be accrued.

(iv) If α is the amount still owing on the loan after 9 years, write an expression involving α for the amount owing after the repayment free period. **1**

(v) Find the new monthly repayment amount, R, which Judy will need to make if she plans to repay the loan in the same amount of time had she not missed any repayments. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2009 HSC MATHEMATICS TRIAL SOLUTIONS

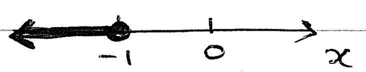
Question 1

a) $\log_e 7 = 1{\cdot}95$ (2dp)

b) $2x + 8 \leq 6$

$2x \leq -2$

$x \leq -1$



c) $\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

d) $\frac{x-2}{x+3} \div \frac{3x-6}{x^2-x-12}$

$= \frac{\cancel{x-2}}{\cancel{x+3}} \times \frac{(x-4)\cancel{(x+3)}}{3\cancel{(x-2)}}$

$= \frac{x-4}{3}$

e) $x + y = 8$ ①

$3x - 2y = -11$ ②

(1a) $x = 8 - y$

(1a) → ② $3(8-y) - 2y = -11$

$24 - 3y - 2y = -11$

$-5y = -35$

$\underline{y = 7}$

into ①: $\underline{x = 1}$

f) $|2x+1| = 7$

$2x+1 = 7$ or $2x+1 = -7$

$2x = 6$ $\quad 2x = -8$

$\underline{x = 3}$ or $\underline{x = -4}$

g) $\frac{5-\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{5\sqrt{3}+5-3-\sqrt{3}}{3-1}$

$= \frac{4\sqrt{3}+2}{2}$

$= 2\sqrt{3}+1$

$\therefore a = 1, \quad b = 2$.

Question 2

a) i) $3y = -7x + 8$

$y = -\frac{7}{3}x + \frac{8}{3}$

$\therefore m = -\frac{7}{3}$.

ii) $m = \tan\theta$

$\therefore \tan\theta = -\frac{7}{3}$ (–ve so obtuse ∠)

$\theta = 113°$ (nearest deg)

iii) $A(-1, 5) \quad B(2, -2)$

$d^2 = (2+1)^2 + (-2-5)^2$

$= 9 + 49$

$\underline{d = \sqrt{58}}$ units.

iv) $m_{AB} = -\frac{7}{3} \therefore m_{AC} = \frac{3}{7}$

(since $m_1 m_2 = -1$ for perp. lines)

iv) cont'd ..

$$y-y_1=m(x-x_1)$$
$$y-5=\frac{3}{7}(x+1)$$
$$7y-35=3x+3$$
$$\therefore \underline{3x-7y+38=0}$$

v) centre $(-1,5)$ radius $\sqrt{58}$

$$\therefore \underline{(x+1)^2+(y-5)^2=58}.$$

vi) $d=\left|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$a=7,\ b=3,\ c=-8\quad x_1=7\quad y_1=-1$

$$\therefore d=\frac{|7(7)+3(-1)-8|}{\sqrt{7^2+3^2}}$$
$$=\frac{38}{\sqrt{58}}\text{ units}$$

vii) $A=\frac{1}{2}bh$

$$=\frac{1}{2}\times\frac{38}{\sqrt{58}}\times\sqrt{58}$$
$$=\underline{19\text{ units}^2}.$$

b) let $e^x=m$

$$\therefore m^2+3m-4=0$$
$$(m-1)(m+4)=0$$
$m=1$ or $m=-4$

$\therefore e^x=1$ or $e^x=-4$ (no solns)

$$\therefore \underline{x=0}.$$

Question 3

a) i) $f(x)=4x\log_e 2x$

$u=4x\qquad v=\log_e 2x$

$u'=4\qquad v'=\frac{2}{2x}=\frac{1}{x}$

$$\therefore f'(x)=uv'+vu'$$
$$=\frac{4x}{x}+4\log_e 2x$$
$$=4+4\log_e 2x$$

ii) $f(x)=\cos(4x-3)$

$f'(x)=-4\sin(4x-3)$

b) $T_3:\quad ar^2=54\quad (1)$

$T_6:\quad ar^5=2\quad (2)$

i) $\therefore\ r^3=\frac{2}{54}=\frac{1}{27}$

$$\underline{r=\frac{1}{3}}$$

ii) sub $\frac{1}{3}\to(1)$: $a(\frac{1}{3})^2=54$

$$a=486$$

$$S_n=\frac{a(r^n-1)}{r-1}$$

$$\therefore S_6=\frac{486\left(\left(\frac{1}{3}\right)^6-1\right)}{-2/3}$$
$$=\underline{728}.$$

c) ii) In ΔMPQ & ΔNRS:

$\angle PMQ = \angle SNR = 90°$ (given)

$\angle PQM = \angle NSR$ (alternate $\angle$s, $PQ \parallel SR$)

$PQ = SR$ (opp sides parallelogram are equal)

$\therefore \Delta MPQ \equiv \Delta NRS$ (AAS).

iii) $PM = NR$ (corresponding sides in congruent Δs from i))

Since $\angle RNS = \angle PMN = 90°$, $PM \parallel NR$ as these are alternate angles.

$\therefore PNRM$ is a parallelogram (1 pair of sides equal & parallel)

Question 4

a) $y = 3e^{2x}$ $(0, 3)$

$\frac{dy}{dx} = 6e^{2x}$

when $x = 0$, $\frac{dy}{dx} = 6e^0$

$= 6$

$\therefore y - y_1 = m(x - x_1)$

$y - 3 = 6x$

$y = 6x + 3$.

b) i) $\int \sec^2 4x \, dx = \frac{1}{4}\tan 4x + c$

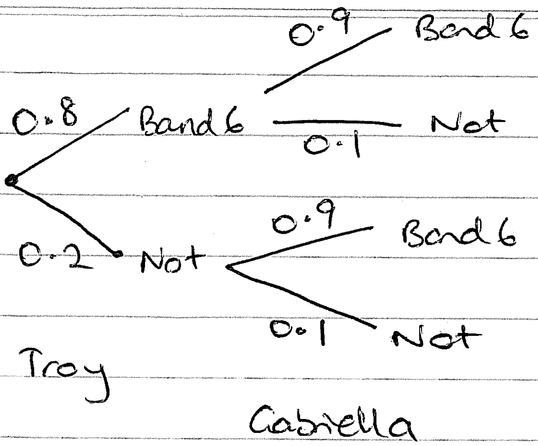
ii) $\int_{-2}^{1} \frac{1}{2-x} dx = -1 \int_{-2}^{1} \frac{-1}{2-x} dx$

$= \left[-\ln(2-x)\right]_{-2}^{1}$

$= -\ln 1 + \ln 4$

$= \ln 4$.

c) i)



ii) P(one fails) = Troy fails × Gab achieves + Gab fails × Troy achieves

$= 0.2 \times 0.9 + 0.1 \times 0.8$

$= 0.26$

iii) P(at least one fails) = 1 − P(no fails)

$= 1 - 0.8 \times 0.9$

$= 0.28$.

d) Positive definite if $a > 0$ and $\Delta < 0$

$\therefore \quad b^2 - 4ac < 0$

$(2-k)^2 - 4(1)(2-k)k < 0$

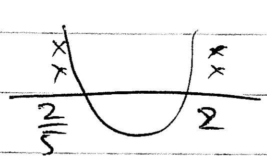
$4 - 4k + k^2 - 8k + 4k^2 < 0$

$4 - 12k + 5k^2 < 0$

$5k^2 - 10k - 2k + 4 < 0$

$5k(k-2) - 2(k-2) < 0$

$(5k-2)(k-2) < 0$



$\therefore \quad \frac{2}{5} < k < 2$.

Question 5

a) $\frac{dy}{dx} = 6(x-1)(x-2)$

i) $\frac{dy}{dx} = 6(x^2-3x+2)$

$\therefore y = 6\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + c$

sub $(1,2)$ $\quad 2 = 6\left(\frac{1}{3} - \frac{3}{2} + 2\right) + c$

$2 = 2 - 9 + 12 + c$

$c = -3$

$\therefore$ eqn is $y = 2x^3 - 9x^2 + 12x - 3$.

ii) SPs occur when $\frac{dy}{dx} = 0$ $\therefore x = 1$ or 2

test: $\frac{d^2y}{dx^2} = 12x - 18$

when $x=1$ $\frac{d^2y}{dx^2} = -6$ $\therefore \cap$ max SP.

when $x=2$ $\frac{d^2y}{dx^2} = 6$ $\therefore \cup$ min SP

find yvalues: when $x=1$, $y = 2-9+12-3 = 2$ so max SP at $(1,2)$

when $x=2$ $y = 2(2)^3 - 9(2)^2 + 12(2) - 3 = 1$ min SP at $(2,1)$

iii) inflexions occur if $\frac{d^2y}{dx^2} = 0$

$12x - 18 = 0$

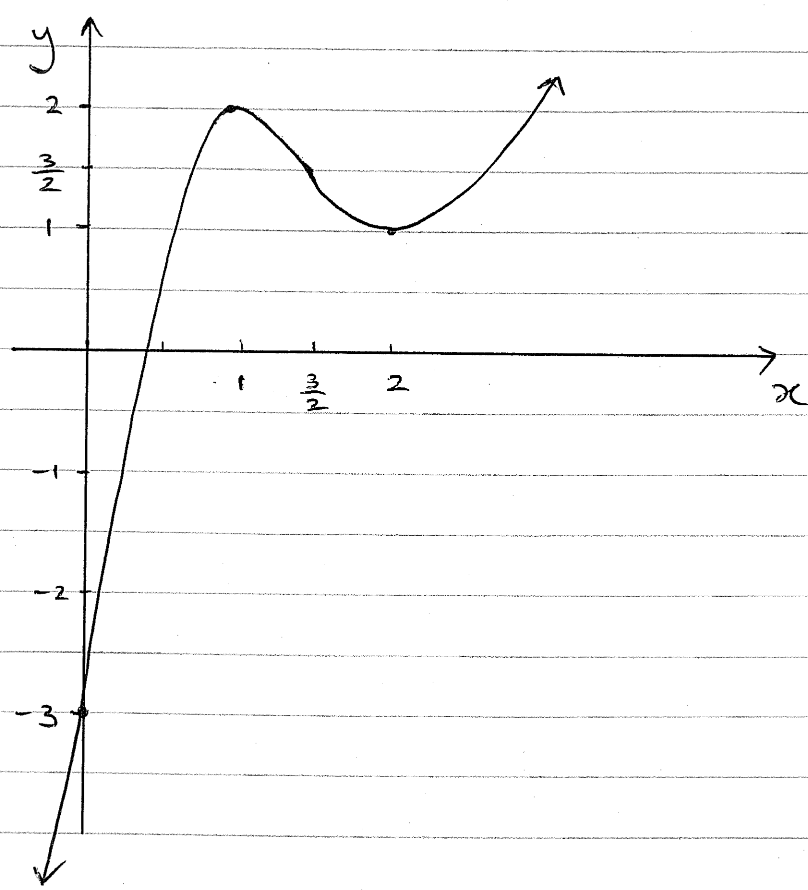
$x = \frac{3}{2}$ $\quad$ find yvalue: $y = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) - 3 = \frac{3}{2}$

test for change in concavity:

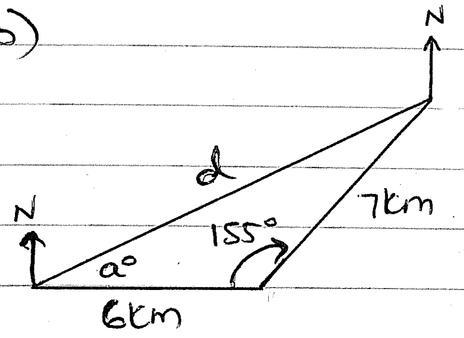
x	1	$\frac{3}{2}$	2
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\therefore \left(\frac{3}{2}, \frac{3}{2}\right)$ is a pt of inflexion.

iv)



b)



i) $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore\ d^2 = 6^2 + 7^2 - 2(6)(7)\cos 155°$

$= 161.129\ldots$

$d = 12.69369\ldots$

$= 12.694$ km (nearest m)

ii) $\dfrac{\sin a°}{7} = \dfrac{\sin 155°}{12.694}$

$\sin a° = 0.233\ldots$

$a° = 13.48°$ (2dp)

$\therefore$ bearing is $(90 - 13.48)°T = 76.52°T$ (2dp)

Question 6

a) i) $l = r\theta$

$6 = 10\theta$

$\theta = \frac{3}{5} \times \frac{180}{\pi}$

$= 34°23'$ (nearest min)

ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(10)^2(\frac{3}{5})$

$= 30cm^2$

iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= 30 - \frac{1}{2}(10)^2(\sin\frac{3}{5})$

$= 1.77cm^2$ (2dp)

b) $1536 + 1472 + 1408 + \dots$

$a = 1536$; $d = -64$

$T_n = 1536 - 64(n-1)$

i) $T_9 = 1536 - 64(8)$

$= 1024$

ii) $64 = 1536 - 64(n-1)$

$64(n-1) = 1472$

$n - 1 = 23$

$\therefore n = 24$ so 24 layers.

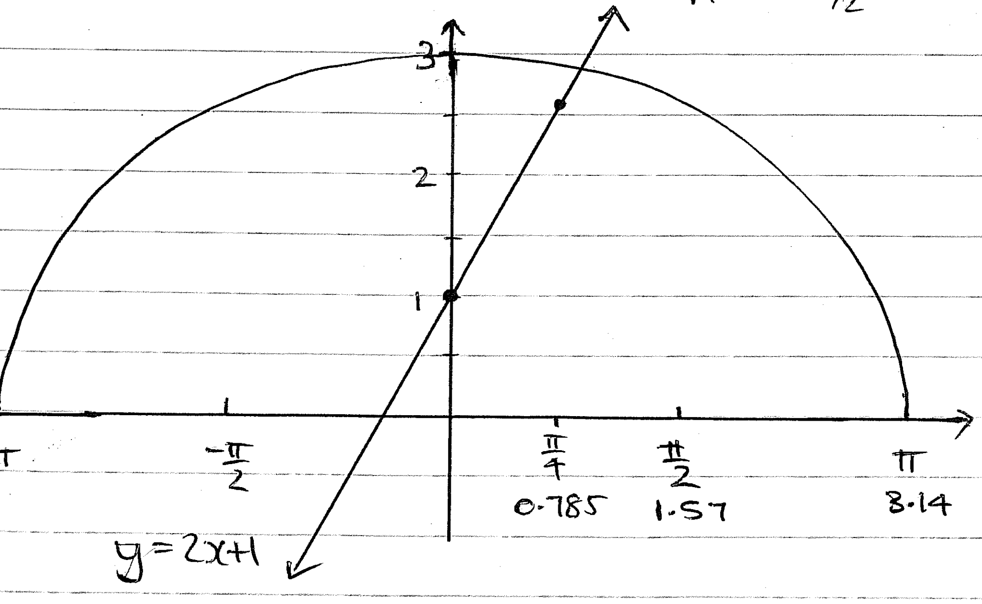
iii) $S_n = \frac{n}{2}(a + l)$

$S_{24} = \frac{24}{2}(1536 + 64) + 1$ //

$= 19201$ blocks.

c) i) $y = 3\cos\frac{x}{2}$ $\quad -\pi \le x \le \pi$.

amplitude $= 3$ $\quad$ period $= \frac{2\pi}{n} = \frac{2\pi}{1/2} = 4\pi$.



ii) to solve $\cos\frac{x}{2} = \frac{2x+1}{3}$, need to sketch $y = 2x+1$

$\therefore$ there is one solution in the given domain.

Question 7

a) i) $y = 2x - 3 \quad (1)$

$y = x^2 - 2x - 3 \quad (2)$

$(1) = (2) \quad 2x - 3 = x^2 - 2x - 3$

$0 = x^2 - 4x$

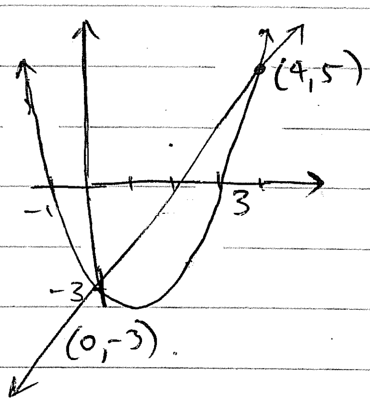
$0 = x(x-4)$

$x = 0$, or $x = 4$,

into (1) $y = -3$ $\qquad y = 2(4) - 3 = 5$

∴ The points of intersection are $(0,-3)$ and $(4,5)$.

ii)



$y = (x-3)(x+1)$

$y = 2x - 3$

$\therefore A = \int_0^4 2x - 3 - (x^2 - 2x - 3)\, dx$

$= \int_0^4 4x - x^2 \, dx$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4$

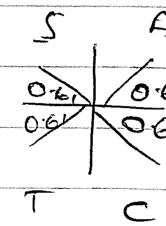
$= 2(4)^2 - \frac{4^3}{3} - (0 - 0)$

$= 10\frac{2}{3}$ units^2

b) $\sec^2 x = \frac{3}{2}$

$\cos^2 x = \frac{2}{3}$

$\cos x = \pm\sqrt{\frac{2}{3}}$



$\therefore x = 0.62,\ 3.76$

$= 2.52 \quad 5.68$

c) $A = xy = 338\text{cm}^2$

$\therefore y = \frac{338}{x}$

i)

$\therefore A_{text} = (x-2)(y-4)$

$= (x-2)\left(\frac{338}{x} - 4\right)$

$= 338 - 4x - \frac{676}{x} + 8$

$= 346 - 4x - \frac{676}{x}$ //.

ii) max/min occurs when $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = -4 + \frac{676}{x^2}$

$\therefore 4 = \frac{676}{x^2}$ when $\frac{dA}{dx} = 0$

$x^2 = 169$

$x = 13$ (dimensions $\neq$ negative)

test in $\frac{dA^2}{dx^2} = -\frac{1352}{x^3}$

when $x = 13$ $\frac{dA^2}{dx^2} = -ve$

∴ max TP ∴ max value

when $x=13$, $y=\frac{338}{13}=26$

$\therefore$ dimensions are $13cm \times 26cm$ to maximise print area.

Question 8

a) i) $AO=OP$ (radii)

so $\angle OAP = \angle OPA$ (equal angles opp. equal sides)

$\angle POB = \angle OAP + \angle OPA$ (exterior $\angle$ ΔAOP)

$\therefore \quad 2x = \angle OAP + \angle OPA$

$\therefore \quad \angle OAP = \angle OPA = x$.

ii) $\sin 2x = \frac{PN}{OP}$ from ΔPON

$= \frac{PN}{\frac{1}{2}AB}$ (since AB is a diameter & OP is a radius)

$= \frac{2PN}{AB}$.

iii) in ΔAPN $\quad \sin x = \frac{PN}{AP}$

in ΔPAB $\quad \cos x = \frac{AP}{AB}$

$\therefore \quad \sin x \cos x = \frac{PN}{AP} \times \frac{AP}{AB}$

$= \frac{PN}{AB}$

$\therefore \quad 2\sin x \cos x = \frac{2PN}{AB} = \sin 2x$ from ii).

//

b) i) $2y = x^2 - 8x + 4$

$2y + 12 = x^2 - 8x + 16$

$2(y+6) = (x-4)^2$

$\therefore$ vertex $(4, -6)$.

ii) using $(x-h)^2 = 4a(y-k)$

$4a = 2 \quad \therefore a = \frac{1}{2}$

$\therefore$ focus is $(4, -5\frac{1}{2})$

directrix is $y = -6\frac{1}{2}$.

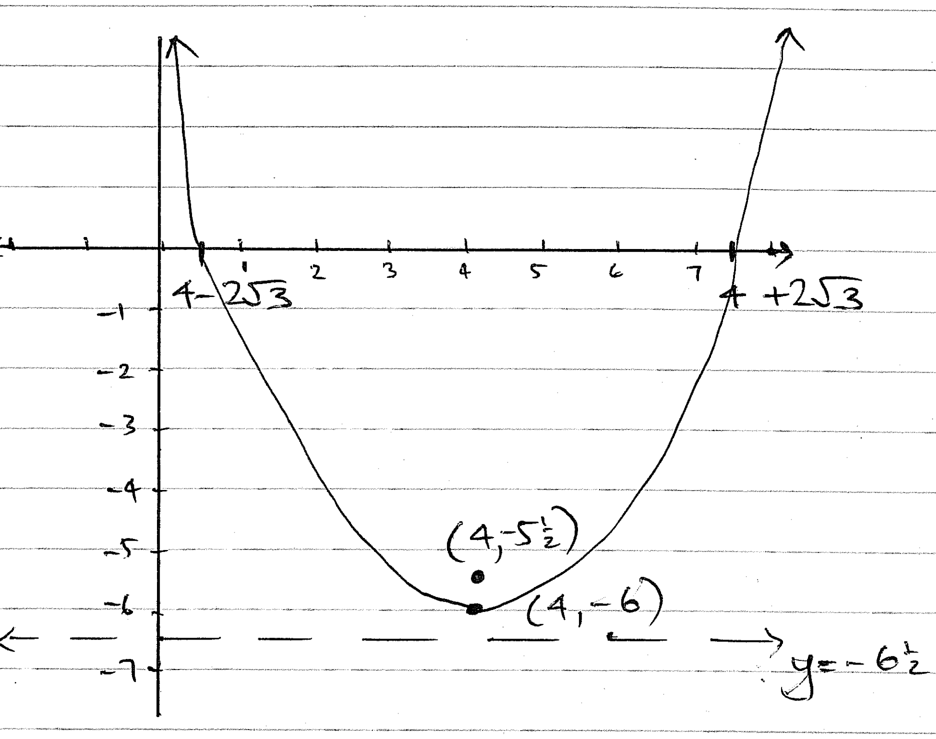
iii) x ints: $y = 0$

$2(0+6) = (x-4)^2$

$12 = (x-4)^2$

$\pm\sqrt{12} = x - 4$

$\therefore x = 4 + 2\sqrt{3}$ or $4 - 2\sqrt{3}$.



Question 9

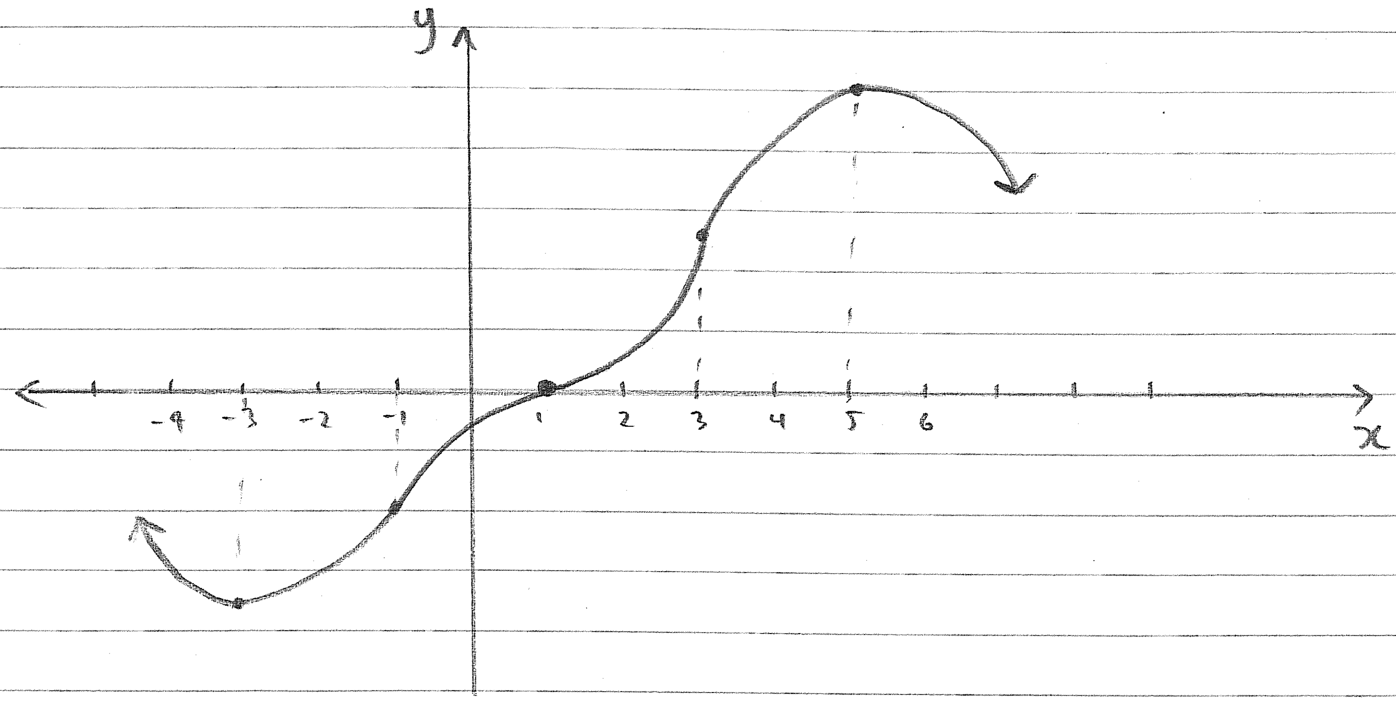
a) $\log_b a \times \log_c b \times \log_a c$

$$= \frac{\log_a a}{\cancel{\log_a b}} \times \frac{\cancel{\log_a b}}{\cancel{\log_a c}} \times \cancel{\log_a c}$$

$= \log_a a$

$= 1$

b)



c) $f(x) = \log_{10} x$

x	1	2	3
$f(x)$	$\log_{10} 1$	$\log_{10} 2$	$\log_{10} 3$

$$\int_1^3 f(x)\,dx = \frac{3-1}{6}\left(\log_{10} 1 + 4\log_{10} 2 + \log_{10} 3\right)$$

$= 0.5604\ldots$

$\doteq 0.560$ (3sf)

d)i) $y = \dfrac{\log_e x}{x}$ $\quad u = \ln x \quad v = x$

$u' = \dfrac{1}{x} \quad v' = 1$

$$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

ii) $V = \pi \int_1^e \left(\frac{\sqrt{\log_e x}}{x} \right)^2 dx$

$= \pi \int_1^e \frac{\ln x}{x^2} \, dx$

from i), $\frac{\ln x}{x^2} = -\left(\frac{1-\ln x}{x^2} - \frac{1}{x^2} \right)$

$\therefore V = -\pi \int_1^e \left(\frac{1-\ln x}{x^2} - x^{-2} \right) dx$

$= -\pi \left[\frac{\ln x}{x} + \frac{1}{x} \right]_1^e$

$= -\pi \left(\frac{\ln e}{e} + \frac{1}{e} - \left(\frac{\ln 1}{1} + 1 \right) \right)$

$= -\pi \left(\frac{2}{e} - 1 \right)$

$= \pi - \frac{2\pi}{e}$ units^3.

Question 10

a) $f(x) = \log_e(1+\sin x)$

$f'(x) = \frac{f'(x)}{f(x)} = \frac{\cos x}{1+\sin x}$ $\qquad u = \cos x \qquad v = 1+\sin x$

$\qquad u' = -\sin x \qquad v' = \cos x$

$f''(x) = \frac{vu' - uv'}{v^2} = \frac{-\sin x(1+\sin x) - \cos^2 x}{(1+\sin x)^2}$

$= \frac{-\sin x - \sin^2 x - (1-\sin^2 x)}{(1+\sin x)^2}$

$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$= \frac{-(\sin x + 1)}{(1+\sin x)^2}$$

$$= \frac{-1}{1+\sin x}$$

b) i) \$300000 ; 6% p.a = 0·005 p.m. R = \$2000 p.m.

$A_1 = 300000(1{\cdot}005) - 2000$

$A_2 = A_1(1{\cdot}005) - 2000$

$\quad = 300000(1{\cdot}005)^2 - 2000(1{\cdot}005) - 2000$

$A_3 = A_2(1{\cdot}005) - 2000$

$\quad = 300000(1{\cdot}005)^3 - 2000(1{\cdot}005)^2 - 2000(1{\cdot}005) - 2000$

∴ after k months

$$A_k = 300000(1{\cdot}005)^k - \underbrace{2000(1{\cdot}005)^{k-1} - \dots - 2000(1{\cdot}005) - 2000}_{\text{G.S with } a = 2000,\ n = k,\ r = 1{\cdot}005}$$

$$A_k = 300000(1{\cdot}005)^k - \frac{2000(1{\cdot}005^k - 1)}{1{\cdot}005 - 1}$$

$= 300000(1{\cdot}005)^k - 400000(1{\cdot}005^k - 1)$

$= 400000 - 100000(1{\cdot}005)^k$

$= 100000\left[4 - (1{\cdot}005)^k\right].$

ii) After 9 years = 108 months : $k = 108$

$\therefore A_{108} = 400000 - 100000(1{\cdot}005)^{108}$

$\quad = 228630{\cdot}0501\ldots$

ie still owes \$228630·05.

iii) Loan repaid when $A_k = 0$

$$100000\left[4 - 1.005^k\right] = 0$$

$$1.005^k = 4$$

$$k \cdot \ln 1.005 = \ln 4$$

$$k = 278 \text{ (nearest payment).}$$

iv) let $\alpha = A_{108}$. Amount owing after interest free period $= A_{126}$;
Interest has been accrued 18 times.

$$\therefore A_{126} = \alpha(1.005)^{18}$$

v) let the new repayment be R.
10 yrs 6 months have passed = 126 payments
$\therefore$ 278 − 126 = 152 payments to go.
$\therefore$ now $B_{152} = 0$ when loan is repaid.

$$B_1 = A_{126}(1.005) - R$$
$$= \alpha(1.005)^{19} - R$$
$$B_2 = B_1(1.005) - R$$
$$= \alpha(1.005)^{20} - R(1.005) - R$$
$$B_3 = B_2(1.005) - R$$
$$= \alpha(1.005)^{21} - R(1.005)^2 - R(1.005) - R$$

$$\therefore B_{152} = \alpha(1.005)^{170} - \underbrace{R(1.005)^{151} - R(1.005)^{150} - \dots - R}$$

G.S. with $a = R$ $\quad r = 1.005$ $\quad n = 152$

$$B_{152} = \alpha(1.005)^{170} - \left[\frac{R(1.005^{152} - 1)}{1.005 - 1}\right]$$

But $B_{150} = 0$

$$\frac{R(1.005^{152} - 1)}{0.005} = \alpha(1.005)^{170}$$

$$R = \frac{\left[400\,000 - 100000(1.005)^{108}\right](1.005)^{170}(0.005)}{(1.005^{152} - 1)}$$

$= 2353.0565\ldots$

$= \$2353.06$

$\therefore$ new repayment amount is $2353.06 //